

Experiment No. 4

## ACTIVE FILTER FREQUENCY RESPONSE

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## **Abstract:**

**Experiment No. 4:** Active Filter Response

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In this lab, a high-pass and a low-pass filter were constructed to decrease the frequency response of an op-amp. The low-pass filter was built first to reduce the high frequency effects on the amplifier. Then the high-pass filter was used to reduce the low frequency effects. Each filter's frequency response was also tested to determine how effective it was.

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## Introduction:

Low-pass and high-pass filters are used to reduce the frequency response of circuits at high frequencies (low-pass) or low frequencies (high-pass). These filters can be used, as there were in the experiment, to reduce the response of an amplifier circuit.

## Background and Theory:

A low-pass filter only allows low frequencies to effect the output, and a high-pass filter only allows high frequencies. For the low-pass filter amplifier in figure 1, the characteristic indicates the gain ( $V_o/V_i$ ) should be maximum for low frequencies, and given by  $R_2/R_1$ . This is because at low frequencies, the capacitor has very high impedance, making  $V_o$  large ( $V_o$  is proportional to the impedance it's across).

The gain begins falling at the cutoff frequency,  $\omega_h$ , which is controlled by  $R_2$  and  $C_2$  by the relationship  $\omega_h=1/(R_2C_2)$ .

At high frequencies, the gain drops off because the capacitor's impedance gets so low; it acts like a short, resulting in  $V_o$  becoming grounded. The gain then drops to zero.

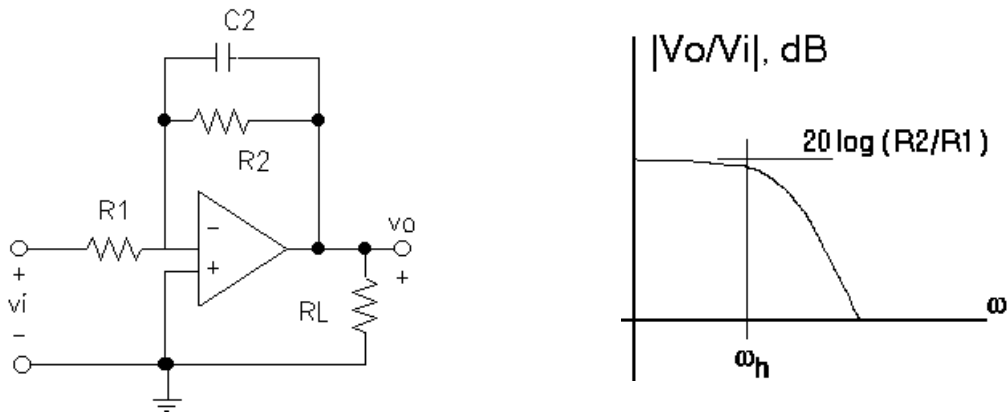


Figure 1 – Low-pass Amplifier

The gain relationship for this circuit is 
$$\frac{V_o}{V_i} = -\frac{R_2}{R_1(1 + j\omega R_2 C_2)} = -\frac{R_2}{R_1 \left(1 + \frac{j\omega}{\omega_h}\right)}$$

A high-pass filter does just the opposite as the low-pass filter. It reduces the low-frequency response of the amplifier circuit. For the high-pass amplifier, as seen in figure 2, the gain equation is:

$$H(\omega) = \frac{V_o}{V_i} = - \left( \frac{R_1 + \frac{1}{j\omega C_1}}{R_2 + \frac{1}{j\omega C_2}} \right) * \left[ -\frac{R_2}{R_1} \right] * \left[ \frac{j\omega C_1}{j\omega C_2} \right] = \left[ -\frac{R_2}{R_1} \right] * \frac{(j\omega C_1 R_1 + 1)}{(j\omega C_2 R_2 + 1)}$$

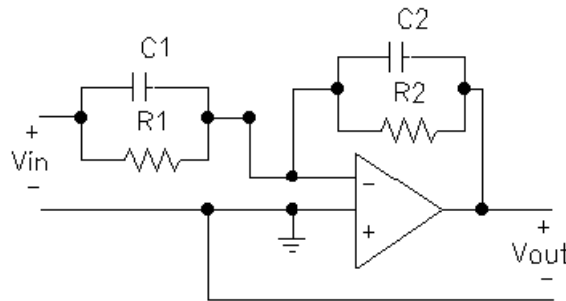


Figure 2 – High-pass Amplifier

From the gain equation it can be seen that a low frequency approximation (which gets better as  $\omega \rightarrow 0$ ) is:

$$H(\omega) = -\frac{R_2}{R_1}$$

And also that a high frequency approximation (which gets better as  $\omega \rightarrow \infty$ ) is:

$$H(\omega) = -\frac{C_1}{C_2}$$

For derivations of the above equations, see Appendices V and VI.

## Procedure:

### Part I: Low-pass Filter

1. Given the ideal values  $R_1=3559 \Omega$ ,  $R_2=11254 \Omega$ , and  $C_1=.001 \mu\text{F}$ , the dc gain for a low-pass filter was calculated to be  $10.0 \text{ dB}$ . The cutoff frequency was calculated to be  $14.14 \text{ kHz}$  (see Appendix V for both calculations).

2. The circuit in figure 1 was constructed using an LM351 op-amp and the following components:

$$R_1: 3.3 \text{ k}\Omega + 270 \Omega = 3.57 \text{ k}\Omega$$

$$R_2: 11 \text{ k}\Omega + 270 \Omega = 11.27 \text{ k}\Omega$$

$$R_L: 4.7 \text{ k}\Omega$$

$$C_1: .001 \mu\text{F}$$

A bipolar power supply set at 15 volts was also used.

3. Using a function generator, a sinusoidal input was used as  $v_i$ . The sinusoid was set for 2 volts peak-to-peak, and  $v_o$  was measured for frequencies within the range of 100 Hz and 1 MHz. Extra attention was given to the range extending from 10 kHz to 100 kHz since this is the range the gain falls through the  $-3 \text{ dB}$  point.

4. The values measured in step 3 were put in a table in Appendix II with the dB value of the gain ( $20 \cdot \log(v_o/v_s)$ ).

5. Another filter like that in figure 1 was constructed and connected in series with the original one. The  $4.7 \text{ k}\Omega$  resistor on the first filter was connected as the input in the second one, and another  $4.7 \text{ k}\Omega$  resistor was used as the output. The completed circuit is a form of the second-order RC filter.

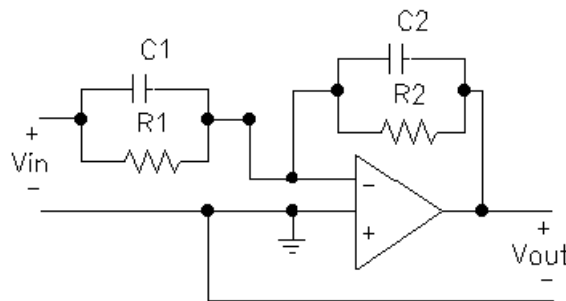
6. The frequency measurements made in step 3 were repeated and recorded in Appendix II. Since the  $-3 \text{ dB}$  frequency has been reduced by 0.6436 (see Appendix III), the careful measurements had to start at a lower frequency than in step 3.

7. The dB gain and  $-3 \text{ dB}$  frequency of an ideal second-order filter were derived and recorded in Appendix IV. The calculated  $f_{-3\text{dB}}$  is  $9.102 \text{ kHz}$ .

## Part II: High-pass Filter

Before beginning this part of the experiment, values for an active high filter (reproduced just below) were found such that the high frequency gain was -5 and the low frequency gain was -1 with a cutoff frequency of 1 kHz. The negative gains were just implying an inverting amplifier. Component values used were within 10% of the calculated values.

See Appendix VI for the calculations of the components.



High-pass Filter Amplifier

1. The filter in the above figure was constructed using the values  $C_1=0.22 \mu\text{F}$ ,  $C_2=0.047 \mu\text{F}$ , and  $R_1=R_2=3.3 \text{ k}\Omega$ .
2. Using a 1 volt peak-to-peak voltage source, the frequency response of the circuit was determined by taking output voltage readings for various frequencies using the oscilloscope. The results were recorded in Appendix II.
3. After shutting off the power, the resistors were swapped with each other, and then the capacitors. The frequency response was rechecked and measurements put in Appendix II. The gain was then just the inverse of what it was originally.
4. The circuit was then used to check the effect of making the ratio of the capacitors equal to the ratio of the corresponding resistors. The gain should've been unity for all frequencies, and taking a few measurements proved this to be true. (See Appendix II for values recorded).

## Results and Analysis:

All results, within reason, are what were expected. A small amount of error may have resulted from error in each components actual value, or from background noise.

In part 1, step 3, the gain calculated for low frequencies (see Appendix II) was approximately the same as that predicted in step 1 (10 dB). For  $f=100$  Hz, the dB gain was 9.883 dB, resulting in an error of only 1.17%. This error was consistent for the lower frequency measurements and was caused by inaccurate equipment or just because the 10 dB prediction was made for a frequency of zero (measurements began at 100 Hz, not zero).

The  $-3$  dB output voltage was measured to be 4.36 volts, a 1.19% error from the predicted value of 4.41 volts (This value results from dividing the voltage measured at 100 Hz by  $\sqrt{2}$ . This may be inaccurate because maximum  $v_o$  may not have occurred at 100 Hz, but at the ideal frequency, zero).

When a second filter was added to create a second order filter in step 5 of part 1, a low frequency gain of 19.913 dB was calculated from measurements made at 100 Hz. This is only a .44% difference than the ideal DC gain of 20 dB. This error resulted again because of inaccurate equipment (like the frequency dial on the function generator).

The  $-3$  dB output voltage was measured to be 14.0, giving an incredibly small error of .0051% from the estimated value of 14.0007 volts.

For the high-pass filter, the following differences were observed:

	Gain from Measurements	Approximated Gain	% Error
Low Frequencies	-1.15	-1	15 %
High Frequencies	-4.48	-4.7	4.68 %

With swapped capacitors and resistors:

	Gain from Measurements	Approximated Gain	% Error
Low Frequencies	-.977	-1	2.3 %
High Frequencies	-.210	-.2	5 %

For  $R_1/R_2 = C_1/C_2$  the error in the gain (which should've been 1, or unity gain) ranged from 5% to 3%. Since this error is consistent, it was probably a result of inaccurate equipment.

## **Conclusion:**

In this experiment the frequency response of an amplifier was reduced using a filter. A low-pass filter was used first to keep frequency response down at high frequencies, and a high-pass filter was then used to keep the response down at low frequencies.

To test the response of the amplifiers with each of the filters, measurements were made at various frequencies. The results showed that for the low-pass filter circuit, the gain drops to zero as the frequency increases, just as expected. For the high-pass configuration, the gain was very close to the approximation for high and low frequencies.

For the low-pass filter, it was seen that for frequencies near zero, the gain was at a max, and directly depended on the resistor values used in the circuit. The frequency at which the gain starts to drop was also dependent of the resistor and capacitor between the negative terminal of the op-amp and  $v_o$ .

For the high-pass filter, the gain for low frequencies could be approximated by the ratio of the resistor values used. The gain for the high frequencies could also be approximated – it was just the ratio of the capacitor values used. It was seen that the gain values for all frequencies could be inverted just by swapping the resistors and capacitors.

## Appendix I – Equipment List:

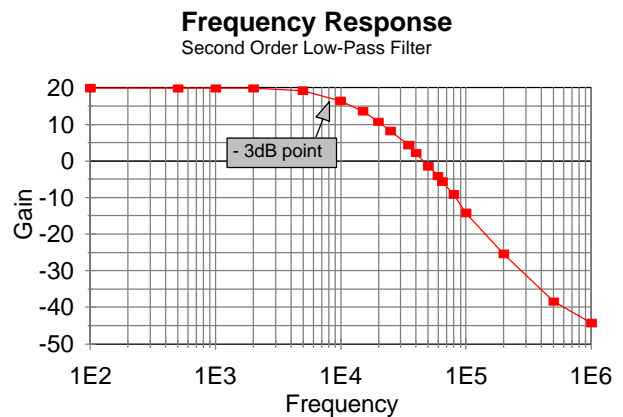
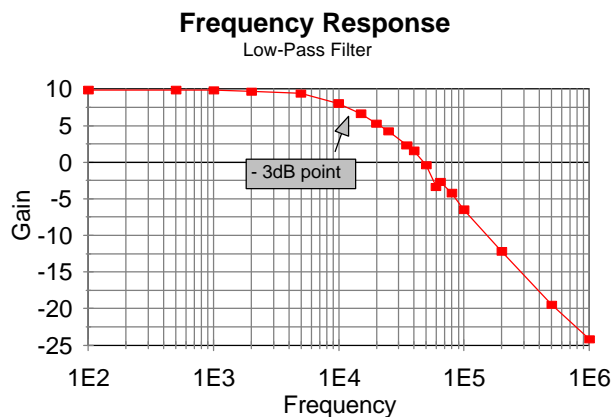
Quant.	Component
2	LM 351 Op-Amp
2	270 $\Omega$ resistor
4	3.3 k $\Omega$ resistor
2	4.7 k $\Omega$ resistor
2	11 k $\Omega$ resistor
3	.001 $\mu$ F capacitor
1	.001 $\mu$ F capacitor
1	bipolar power supply
1	function generator
1	oscilloscope
1	digital multimeter
various	coax connectors w/ alligator clips

## Appendix II – Measured Voltages and Gains:

### Part I, steps 4 and 6 – Low-pass filter:

$v_{in}=2.0$  volts peak-to-peak

Frequency (Hz)	Output voltage for Step 4 (volts)	Gain for single order, Step 4 (dB)	Output voltage for Step 6 (volts)	Gain for second order, Step 6 (dB)
100	6.24	9.883	19.8	19.913
500	6.21	9.841	19.7	19.869
1000	6.19	9.813	19.7	19.869
2000	6.07	9.643	19.7	19.869
5000	5.89	9.382	18.2	19.181
10000	5.03	8.011	13.3	16.456
15000	4.29	6.629	9.55	13.579
20000	3.68	5.296	6.88	10.731
25000	3.26	4.244	5.12	8.165
35000	2.61	2.312	3.3	4.350
40000	2.39	1.547	2.57	2.178
50000	1.91	-0.400	1.7	-1.412
60000	1.36	-3.350	1.25	-4.082
65000	1.47	-2.674	1.05	-5.597
80000	1.23	-4.222	0.699	-9.131
100000	0.949	-6.475	0.392	-14.155
200000	0.492	-12.181	0.108	-25.352
500000	0.213	-19.453	0.0242	-38.344
1000000	0.124	-24.152	0.0123	-44.222



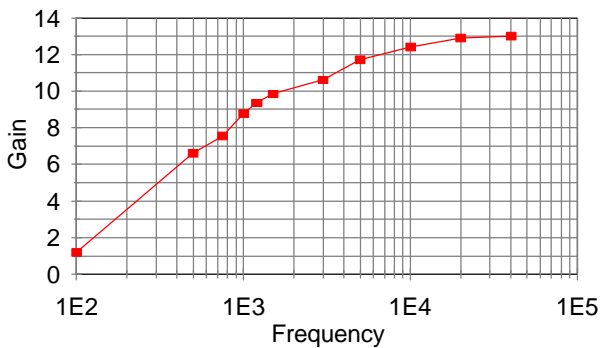
**Part II, steps 2 and 3 – High-pass filter:**

$v_{in}=2.0$  volts peak-to-peak

Frequency (Hz)	Output voltage for Step 2 (volts)	Gain for Step 2 (dB)	Output voltage for Step 3 (volts)	Gain for reversed components Step 3 (dB)
100	-1.15	-4.807	-0.977	-6.223
500	-2.14	0.588	-0.516	-11.768
750	-2.39	1.547	-0.393	-14.133
1000	-2.75	2.766	-0.323	-15.837
1200	-2.94	3.346	-0.308	-16.250
1500	-3.11	3.835	-0.267	-17.490
3000	-3.4	4.609	-0.246	-18.202
5000	-3.86	5.711	-0.227	-18.900
10000	-4.18	6.403	-0.221	-19.133
20000	-4.42	6.888	-0.213	-19.453
40000	-4.48	7.005	-0.21	-19.576

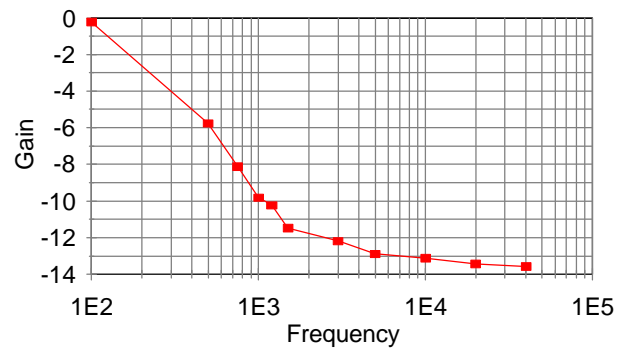
**Frequency Response**

High-Pass Filter



**Frequency Response**

High-Pass Filter w/ Swapped Components



**Part II, step 4 – High-pass filter with component value ratios equal:**

Frequency (Hz)	Output voltage for Step 4 (volts)	Gain for Step 4
100	-1.05	-1.05
500	-1.04	-1.04
750	-1.03	-1.03
1000	-1.06	-1.06
1200	-1.04	-1.04
1500	-1.03	-1.03
3000	-1.03	-1.03



### Appendix III – Derivation of the 0.6436 Factor:

In step 6 of Part 1, the factor of 0.6436 was mentioned. It was derived as follows:

For a single order circuit, the gain is: 
$$H(\omega) = \frac{R_2}{R_1 \left(1 + \frac{j\omega}{\omega_h}\right)}$$

For a second order circuit like that studied in part 6, the gain is just the square of this, or:

$$H_2(\omega) = \left( \frac{R_2}{R_1 \left(1 + \frac{j\omega}{\omega_h}\right)} \right)^2 = \left( \frac{R_2}{R_1} \right)^2 \frac{1}{\left(1 + \frac{j\omega}{\omega_h}\right)^2}$$

Since the maximum of this gain is  $(R_2/R_1)^2$ , the –3 dB frequency would occur at:

$$H_2(\omega_{-3dB}) = \frac{\left(\frac{R_2}{R_1}\right)^2}{\sqrt{2}} = \left(\frac{R_2}{R_1}\right)^2 \frac{1}{\left|1 + \frac{j\omega}{\omega_h}\right|^2} \quad \text{Or just,} \quad \frac{1}{\sqrt{2}} = \frac{1}{\left|1 + \frac{j\omega}{\omega_h}\right|^2}$$

After carrying out the square on the right side of this relationship, the following expression is gotten, and the magnitude can be taken as follows:

$$\frac{1}{\sqrt{2}} = \frac{1}{\left|1 + \frac{j2\omega}{\omega_h} - \left(\frac{\omega}{\omega_h}\right)^2\right|} = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_h}\right)^2\right)^2 + \left(\frac{2\omega}{\omega_h}\right)^2}} = \frac{1}{\sqrt{1 - 2\left(\frac{\omega}{\omega_h}\right)^2 + \left(\frac{\omega}{\omega_h}\right)^4 + \left(\frac{2\omega}{\omega_h}\right)^2}}$$

Squaring both sides and taking the inverse, as well as collecting like powers of  $\omega$  results in:

$$2 = 1 - 2\left(\frac{\omega}{\omega_h}\right)^2 + \left(\frac{\omega}{\omega_h}\right)^4 + \left(\frac{2\omega}{\omega_h}\right)^2 = \left(\frac{\omega}{\omega_h}\right)^4 + (2^2 - 2)\left(\frac{\omega}{\omega_h}\right)^2 + 1$$

Multiplying through by  $(\omega_h)^4$  and putting into a quadratic form,

Now, solving the quadratic  $(w^4 - 2w^2(w_h)^2 + (w_h)^4) = 0$

$$w^2 = \frac{-2(w_h)^2 \pm \sqrt{4(w_h)^4 + 4(w_h)^4}}{2} = -(w_h)^2 \pm \frac{2\sqrt{2}}{2}(w_h)^2 = (\pm\sqrt{2} - 1)(w_h)^2$$

Since  $w$  must be positive and real, the only logical solution to this is:

$$w = \sqrt{\sqrt{2} - 1}(w_h) \cong .6436(w_h)$$

**Therefore,  $w$  (the  $-3$  dB frequency for the second order configuration) has been reduced from its original value,  $w_h$ .**

## Appendix IV – Ideal Second-Order Low-pass Filter:

### Calculations for Step 7 of Part 1

Given the component values  $R_1=3.559 \text{ k}\Omega$ ,  $R_2=11.254 \text{ k}\Omega$ , and  $C_1=0.001 \text{ }\mu\text{F}$ , the gain for an ideal second-order low-pass filter is:

$$H_2(\omega) = \left( \frac{R_2}{R_1 \left( 1 + \frac{j\omega}{\omega_h} \right)} \right)^2 = \left( \frac{R_2}{R_1} \right)^2 \frac{1}{\left( 1 + \frac{j\omega}{\omega_h} \right)^2} = \frac{10.0(126.6525 * 10^{-12})}{(11.254 * 10^{-6} + j\omega)^2}$$

(this result was obtained by first multiplying through by  $(\omega_h)^2$ )

The dB gain would then be:

$$20\log(10) + 20(2)\log\left(\frac{1}{1 + j(11.254 * 10^{-6})\omega}\right) = 20 + 40\log\left(\frac{1}{1 + j(11.254 * 10^{-6})\omega}\right) \text{ dB}$$

The **DC Gain** would then be at  $\omega=0$ , when the gain is **20 dB**.

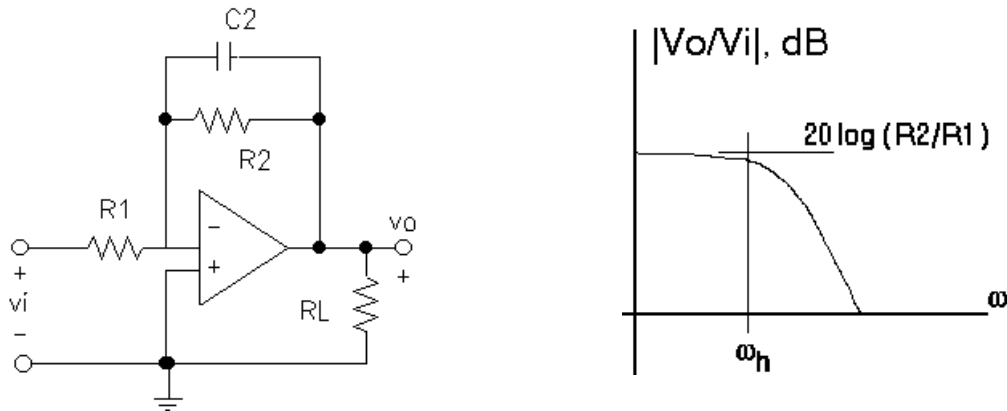
The new  $-3 \text{ dB}$  frequency can be found by multiplying  $\omega_h$  by the factor derived in Appendix V (.6436)

$\omega_h=1 / (R_2C_1) = 88.8573$  which implies a frequency of  $f=14.14208 \text{ kHz}$ .

Therefore, the frequency for the second order circuit must be:

$$f_2 = 14.14208 * (.6436) = \mathbf{9.102 \text{ kHz}}$$

## Appendix V – About the Low-pass Filter:



The voltage gain for this low-pass amplifier can be derived by first writing a KCL equation at the (-) input of the op-amp:

$\frac{V_i}{R_1} + \frac{V_o}{R_2} + j\omega C_2 V_o$  And this implies that the gain ( $V_o/V_i$ ) is:

$$\frac{V_o}{V_i} = -\left(\frac{R_2}{R_1}\right) \frac{1}{1 + j\omega C_2 R_2} = -\left(\frac{R_2}{R_1}\right) \frac{1}{\left(1 + j\frac{\omega}{\omega_h}\right)}$$

Where  $\omega_h$  is the - 3 dB (half-power) frequency.

With the values given in lab, the -3 dB frequency could be solved for:

$R_1=3559 \Omega$ ,  $R_2=11254 \Omega$ , and  $C_1=.001 \mu\text{F}$  , therefore,

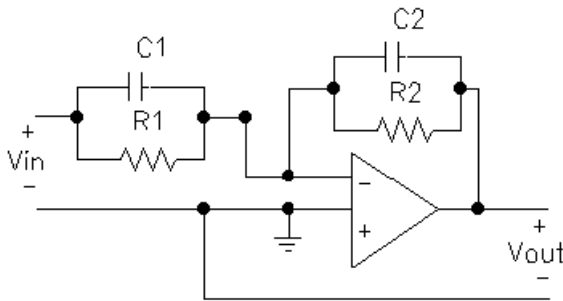
$$\omega_h = \frac{1}{R_2 C_2} = 88.86 \text{ rads/sec}$$

Or  $f = \omega_h/2\pi = 14.14 \text{ kHz}$

The DC gain occurs when  $\omega = 0$  is used in the above gain equation:

DC Gain =  $-R_2/R_1 = -3.16$  or  $20 \cdot \log(|-3.16|) = 10.0 \text{ dB gain}$

## Appendix VI – About the High-pass Filter:



In Part II, the above circuit had to be designed to give a high frequency gain of  $-1$ , a high frequency gain of  $-5$ , and a cutoff frequency of  $1$  kHz. To find the gain, a KCL equation was written at the negative (-) input of the op-amp resulting in:

$$H(\omega) = \frac{-Z_2}{Z_1} = \frac{\left[ \frac{-R_2}{j\omega C_2} \right]}{\left[ \frac{R_2 + \frac{1}{j\omega C_2}}{\frac{-R_2}{j\omega C_2}} \right]} = \left[ \frac{-R_2}{j\omega C_2} \right] \left[ \frac{R_2 + \frac{1}{j\omega C_2}}{-R_2} \right] = \left[ \frac{-R_2}{R_1} \right] \left[ \frac{j\omega R_1 C_1 + 1}{j\omega R_2 C_2 + 1} \right]$$

Therefore, the low frequency approximation (when  $\omega$  is close to zero) is:

$$H(\omega) = -\frac{R_2}{R_1}$$

And the high frequency approximation (when  $\omega$  is very large) is:

$$H(\omega) = -\frac{C_1}{C_2}$$

To find  $R_1$ ,  $R_2$ ,  $C_1$ , and  $C_2$  the frequency approximations were fixed as follows:

$$-\frac{R_2}{R_1} = -1 \quad \text{Therefore, } R_1 = R_2$$

$$-\frac{C_1}{C_2} = -5 \quad \text{Therefore, } C_1 = 5C_2.$$

The resistor values picked were 3.3 kΩ (just so they were greater than 1 kΩ). However, to find the capacitor values, the 1 kHz cutoff frequency results in the following relationship (see the above gain derivation). This is for the definition of the gain at the -3 dB point.

$$\left[ \frac{H(\infty)}{\sqrt{2}} \right] = - \left( \frac{R_2}{R_1} \right) \left[ \frac{\sqrt{(wR_1C_1)^2 + 1^2}}{\sqrt{(wR_2C_2)^2 + 1^2}} \right] = \left| \frac{\left( - \frac{R_2(R_1C_1)}{R_1(R_2C_2)} \right)}{\sqrt{2}} \right| = \frac{5}{\sqrt{2}}$$

$$w = 2\pi f = 2\pi(1k) = 6.283$$

This can be manipulated to give another equation relating C<sub>1</sub> and C<sub>2</sub>:

$$\frac{25}{2} = 1^2 \frac{(w_{-3db}R_1C_1)^2 + 1^2}{(w_{-3db}R_2C_2)^2 + 1^2} = \frac{((6.283k)(3.3k)5C_2)^2 + 1}{(6.283k)((3.3k)C_2)^2 + 1}$$

Which implies:

$$\frac{25}{2} + 5.374 * 10^{15} (C_2)^2 = 1 + 10.75 * 10^9 (C_2)^2$$

And solving for C<sub>2</sub>:

$$C_2 = .231 \mu\text{F} \approx .22 \mu\text{F}$$

$$\text{And from this, } C_1 = .0463 \mu\text{F} \approx .047 \mu\text{F}$$

The capacitor values used were .22 μF and .047 μF which were within 10% of the calculated values (.22 μF results in an error of 4.8%, and .047 μF in an error of only 1.5%).

Therefore the all values used were:

R <sub>1</sub>	3.3 kΩ
R <sub>2</sub>	3.3 kΩ
C <sub>1</sub>	.047 μF
C <sub>2</sub>	.22 μF

## **References:**

The author created all graphs and figures.