Semester Project

Building an Equalizer

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EE262 A
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Abstract:

Lab 262 semester Project: Building an Equalizer

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Introduction:

The MATLAB functions `ctft()` and `ctift()` are used to take the Fourier transform and inverse transform of a signal, respectively. If desired, `ctft()`can also be used to plot the signal's amplitude and phase spectrum the signal, and `ctift()`used to plot the signal, given its transform. With these useful functions which numerically solve Fourier transform problems, the Fourier transform theorems can be proven, as was done with a couple theorems in this lab.

Problems may occur with these functions when using truncated signals, so the `ctft()` function will be tested with truncated signals to determine the effects.

Background and Theory:

The Fourier transform can be used to find the amplitude and phase spectrum for periodic energy signals. The transform, *X(f)*, of a signal *x(t)* is defined as

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2pft} dt$$

And the inverse transform as

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2pft} df$$

X(f) is basically just the frequency domain representation of *x(t)*, which is in the time domain.

The amplitude spectrum |*X(f)*| is the amplitude of the frequency domain representation of *x(t)* at various frequencies, and the phase spectrum is $\angle X(f)$.

The functions numerically evaluate to above integrals, making it possible to find the transform of very complex functions, or even just functions that would be hard to integrate when multiplied by an exponential. However, for MATLAB to be able to evaluate the transform, the signal must have a finite length (a computer can't evaluate a signal of infinite length, since it would have to sum up an infinite number of samples).

For this reason, signals must be truncated before using `ctft()` or `ctift()`. Truncating a signal is the same as multiplying it by a rectangular pulse:

The truncated signal, $x_t\,(t)$, of the signal *x(t)* is:

$$x_t\,(t) = \prod\left(\frac{t - t_a}{t}\right)x(t)$$

What this is implying is that the signal is zero for all *t* except between the range of $t = t_a - t\,/\,2$ and $t = t_a + t\,/\,2$. In this range, the signal takes its normal value.

Using the convolution theorem and the transform of a rectangular pulse, the resulting spectrum that MATLAB takes samples of is:

$$X_t\,(f) = X(f)\left(t\ \mathrm{sin}c(tf)e^{-j2pft_a}\right)$$

An undesirable consequence to using the truncated signal is the distortion caused. The 'truncation distortion' increases as the interval τ decreases (the truncation becomes less and less like the actual interval). If $\tau \rightarrow \infty$, the sinc function above becomes an impulse function, which then makes X_τ equal to X(f), which is what was originally desired. This can be expected because by letting $\tau \rightarrow \infty$, the interval is correctly being represented.

Relationships of *x(t)* and *X(f)* include the facts that for real signals, $|X(f)|=|X(-f)|$ and $\angle X(-f) = -\angle X(f)$ (note that this implies $X(-f) = X^*(f)$). If the signal is even, X(f) is both real and even (the total spectrum can then be plotted, instead of just the phase and amplitude).

NOTE: The functions used in MATLAB, `ctft()`and `ctift()`, are expounded upon in Appendix I.

Procedure, Results, and Analysis:

Refer to Appendix II for all MATLAB commands.

1. The Fourier transform of the function $x(t) = \prod\left(\frac{t-1}{2}\right)$ was taken using `ctft()` on the interval $-1 < t < 3$ with a spacing of 0.01 and a maximum spectrum signal spacing of 0.02. The spectra plotted with `ctft()` went from $-2 < f < 2$. The actual statement used was:

```
T=.01;
t=-1:T:3;
x=4*(t>=0 & t<=2);   % Produces the pulse function
ctft(t,x,.02,1,0,-2:2,0:2:8,[.1,.1,.8,.4],-
4:2:4,[.1,.6,.8,.4],'X(f)','|X(f)|','angle(X(f))');
```

See Appendix I for the `ctft()` syntax.

Along with this plot, the analytically derived spectra (see Appendix III) were plotted. Given the spectra, `x2=4*2*sinc(2*f).*exp(-j*2*pi*f)`, the phase and magnitude were graphed using the following commands:

```
magx2=abs(x2);   % Takes the amplitude of x2
phasex2=angle(x2);   % Takes the phase of x2
plct(f,magx2,-2:2,0:2:8,[.1,.1,.8,.4],1,1.5,'-',0);
plct(f,phasex2,-2:2,-4:2:4,[.1,.6,.8,.4],1,1.5,'-.',0);
```

See Appendix IV for the `plct()` syntax.

`ctift()` was then used to take the inverse transform of the derived expression and plot it with the original given *x(t).* The following commands were used:

```
ctift(f,x2,.02,1,1,-2:2,-5:5:5,[.1,.1,.8,.8],-
2:2,[.1,.1,.8,.8],'x(t)','','');
plct(t,x,-2:2,-5:5:5,[.1,.1,.8,.8],1,1.5,'--',0);
```

See Appendix I for the `ctift()` syntax.

There were discrepancies between the actual function and the one plotted using the inverse transform. This is because of ‘Gibb’s Phenomena’.

2. To study the effect of truncating a signal, the signal $x(t) = 2e^{-3|t|}$ was truncated for the three intervals $-2 < t < 2$, $-1 < t < 1$, and $-0.5 < t < 0.5$ with a sample spacing of 0.01, and plotted. The spectrum was found and plotted for $-3 < f < 3$ using a sample spacing of 0.02. The analytically derived spectrum (see Appendix III) was also plotted over the same interval. To do this entire step, an m-file was created and named `Lab6_1.m.` The syntax for this function is as follows:

```
function Lab6_1(s,e)

% Function for Lab 6 part 2. Does everything needed.
% s = start of interval, e = end of interval
```

The function does everything just like in step 1, except that since the signal was even and real, the whole spectrum could be plotted. See Appendix II for the complete function. See Appendix V for output plots and graphs.

It can be seen from the output that as the truncation become greater (the interval decreased), there was more and more error in the transform, just as was predicted in the theory section.

3. The signal $x(t) = 3e^{-2|t|}u(t)$ was used to study the time-shift and modulation theorems for Fourier transforms. The known spectrum is derived in Appendix III.

   a. The signal was plotted using `plct()` over *–1 < t < 7* using a sample spacing of 0.02. The spectrum was also plotted over the interval *–2 < f < 2* with a spacing of 0.02.

      See Appendix V for output.

   b. Part *a* was repeated after shifting the signal by 0.25. The following expression was used to define the new signal:

      ```
      y=3*exp(-2*(t+0.25)).*us(t+0.25);
      ```

      After repeating step *a*, the phase spectrum of the original spectrum and the phase spectrum caused by a time shift were plotted on the same set of axes as well as the sum of the two. From the output in Appendix V, it can be seen that the computed phase spectrum and known spectrum match up. The time shift just resulted in a linear phase spectrum addition.

   c. To demonstrate double-sided amplitude modulation, the original signal was multiplied by the carrier signal cos(10πt). The new signal was defined by this command:

      ```
      xnew=x.*cos(10*pi*t);
      ```

      The `.*` had to be used because the two matrixes, `x` and `cos(10*pi*t)` had to have *each matrix element* multiplied by the corresponding one in the other matrix (like a dot product). This operation was used for all matrix multiplications made.

Step *a* was repeated for this new function.

The amplitude characteristics found in Appendix V should be proportional to *1/f*.

The phase spectrum characteristics are actually about the same as the computed phase spectrum, or a straight line proportional to *f*.

The computed and known amplitude spectra have a lot of difference, but this is caused again by the truncation. Modulating a truncated signal appears to make this error greater. This could also be by the low carrier frequency (5 Hz). Typically the carrier frequency is around 1 MHz.

Conclusion:

In this lab, the `ctft()` and `ctift()` were found to be useful in numeric evaluation of Fourier transforms and inverse transforms. They eliminate the need to evaluate difficult integrals that often arise in Fourier transform calculations (unless a symbolic result is wanted).

These functions *were* somewhat inaccurate when using truncated signals, but this was discussed in the background and theory, and error was expected. It was proven that the smaller the interval was (the greater the truncation), the more error there was. Despite cases with truncated signals, derived transforms (derived by definition) clearly proved the validity of these two functions.

The time-shifting and modulation properties for Fourier transforms were also investigated. The time-shifting property was shown to be correct and accurate, but the modulation theorem caused error, but these were determined not to be serious.

Appendix I – The `ctft()`and `ctift()`function